1. INTRODUCTION

It is estimated that, on average, companies produce three barrels of water for each barrel of oil [1], seventy percent of which comes from weakly consolidated or unconsolidated sandstone. The intrusion of formation water into water-wetted but oil-saturated sand, which is the usual case in oil fields, may trigger or worsen the sand instability that has been frequently observed both in the field [2, 3] and in the laboratory [4-6]. Some characteristics of water-related sand production are:

- Sands become unstable and start to flow after water intrusion even though no preceding sand production was observed [3,4], and massive sand production occurs when $S_w$ reaches a particular value [6];
- For sanding wells, the average sanding rate during water breakthrough is higher than before breakthrough [2];
- The critical global pressure gradient that activates sanding drops when $S_w$ increases [5]; and,
- Sanding appears as an episodic phenomenon: at a given $S_w$, a sand cavity starts to grow and then becomes stabilized; additional cavity growth episodes require either an increase of pressure gradient or a change in the water saturation value [5,6].

Extensive experiments have been carried out to study the effect of changes in $S_w$ (or moisture content, humidity, etc.) on different rock samples, such as shale [7, 8], chalk [9-11], and sandstone [2, 5, 12-15]. The various possible mechanisms may be generalized as follows [16]:

- Chemical reactions between water and solids and the dissolution of cementitious materials may weaken the rock;
- Changes in the surface tension and capillary force may lower the cohesive strength;
• A higher pressure gradient may develop since the relative permeability of oil is decreased with an increase in $S_w$; therefore there is a higher fluid velocity and drag force (seepage force) that may destabilize the sand; and,
• Particles plucked out of the rock skeleton by fluid flow and the swelling of clay materials may block pore throats and locally increase the pressure gradient and thus increase the destabilizing force.

Although the influence of water influx and $S_w$ on sand stability has been appreciated for several decades, quantitative models have proven difficult, as compared to single-phase frictional sand production models without capillarity. In this paper, a comprehensive study of why sand fails after water breakthrough is carried out. This is based on experimental findings and a series of mathematical models, including a rock strength model that considers strength weakening due to both the effects of chemical reactions and capillarity changes, a fluid pressure model based on micromechanics of a biphasic fluid environment, a coupled analytical elastoplastic model for stress estimation, and an improved nonlinear model for rock deformation modulus.

This article is one of a series of studies, and it focuses more on model results to reveal the physics involved in water-related sand production. More details of the developed models can be found in other publications [17-20].

2. MODEL DEVELOPMENT

2.1. Strength model

Although there may be several physical and chemical processes involved [16], the general trend is that an increase in water saturation reduces rock strength, to the extent of 8% [12] to 98% [13], depending on rock texture, mineralogy, fluid chemistry, time, etc.

While some still doubt the significance of capillary effect on rock stability [21], most believe that capillarity plays an important role in sand production after water breakthrough into an oil well [3,6,9]. Consider two identical particles contacting tangentially (Fig. 1); the capillary force ($F_c$) in the liquid menisci between particles can be expressed as (a Nomenclature follows the article)

\[ F_c = \pi x_p^2 \Delta P \quad (1) \]

Assuming the shape of the liquid bridge is a toroid, the capillary pressure ($\Delta P$) is

\[ \Delta P = \gamma \left( \frac{1}{x_p} - \frac{1}{r} \right) \quad (2) \]

Using a Mohr-Coulomb (M-C) strength criterion, the unconfined compressive strength ($\sigma_{UCS}$) can be approximated as

\[ \sigma_{UCS} = 2 \left( 1 - \phi \right) \frac{F_c}{\phi} \frac{x_p}{1 - \sin \phi} \frac{R^2}{R^2} \quad (3) \]

where $\phi$ is the graphical M-C friction angle, $\phi$ is rock porosity, and $R$ is the radius of the spheres. Thus, the rock strength is related to rock porosity, friction angle, particle size and capillary force. Notice that in the equation the only variable changing with water saturation is $F_c$. With an assumption of zero contact angle, the water saturation $S_w$ can be related to the geometrical parameters through [18]

\[ \eta S_w = -\frac{\alpha_w}{2} + \frac{1}{2} \tan \alpha_w + \left( \frac{\alpha_w}{2} - \frac{\pi}{4} \right) \frac{1 - \cos \alpha_w}{\cos \alpha_w} \quad (4) \]

where $\eta$ is a factor accounting for non-uniform particle size effects on total rock strength [22].

Chemical reactions, such as quartz hydrolysis reducing silica-to-hydrogen bond energy, carbonate cement dissolution that physically changes the shape and size of cementitious deposits, new ferruginous mineral deposition, clay swelling, etc. are too difficult to be rigorously quantified with
respect to sand stability [16]. An empirical approach using a time-exponential relationship is recommended for incorporating chemically reduced strength ($C_{o,ch}$) into stress calculations

$$C_{o,ch} = a \exp(-bt)$$  

(5)

where $a$ and $b$ are coefficients determined through curve fitting and $t$ is time.

### 2.2. A pressure model at the microscopic level

From Fig. 2, the force from the pore fluid pressure acting on the particle surface is

$$P(r)A = P_w(r)A_w + P_o(r)A_o$$  

(6)

where subscripts $w$, $o$ represent water phase and oil phase, respectively, $P$ is an apparent fluid pressure at distance “$r$” from the wellbore and for use in the effective stress equations derived later, and $A$ is the particle surface. The ratio of $A_w/A$ and $A_o/A$ can be derived within the dashed frame of Fig. 2 as

$$A_w/A = 2\alpha/\pi, \quad A_o/A = 1 - 2\alpha/\pi$$  

(7)

The difference of the two fluid pressures ($P_w$ and $P_o$) is equal to the capillary pressure:

$$P_w - P_o = -P_c$$  

(8)

Therefore, Eq. (6) becomes

$$P(r) = P_w(r) + P_c(1 - \frac{2\alpha}{\pi})$$  

(9)

or

$$P(r) = P_o(r) - P_c \frac{2\alpha}{\pi}$$  

(10)

Assuming steady-state fluid flow in an infinite reservoir, oil and water pressures are

$$P_w(r) = P_2 - \frac{Q_w \mu_w}{2\pi kk_w h} \ln \left( \frac{R_2}{r} \right)$$  

(11)

$$P_o(r) = P_2 - \frac{Q_o \mu_o}{2\pi kk_o h} \ln \left( \frac{R_2}{r} \right)$$  

(12)

where $P_2$ is far-field flowing pressure at distance $R_2$, $h$ is reservoir thickness, $k$ is absolute permeability; $k_r$ is fluid relative permeability, $Q$ is the fluid flow rate assumed, and $\mu$ is fluid viscosity. Substituting the pressures into Eq. (9), with constant total production rate ($Q = Q_w + Q_o$), an expression for pore pressure can be written as

$$P(r) = P_2 - \frac{Q_\xi(S_w)}{2\pi k h} \ln \left( \frac{R_2}{r} \right)$$  

(13)

where

$$\xi(S_w) = \frac{2\alpha}{\pi} \frac{f_w}{k_{rw}/\mu_w} + \left(1 - \frac{2\alpha}{\pi}\right) \frac{f_o}{k_{ro}/\mu_o}$$

Furthermore, $f_w$ and $f_o$, water and oil fractions in fluid production respectively, can be related to each other through $f_w = 1 - f_o$. The water fraction is then calculated through

$$f_w = \frac{Q_w}{Q} = \frac{-\frac{Ak_{rw}}{\mu_w} \frac{dP_w}{dr}}{-\frac{Ak_{rw}}{\mu_w} \frac{dP_w}{dr} + \frac{k_{ro}}{\mu_o} \frac{dP_o}{dr}}$$  

(14)

Considering capillary pressure to be only related to water saturation (i.e. $dP_c/dr = 0$), the above equation becomes

$$f_w = \frac{1}{1 + \frac{k_{rw}\mu_w}{k_{ro}\mu_o}}$$  

(15)

Since the value of the water volume angle $\alpha$ is related to water saturation through Eq. (4), there will be a specific value of pore pressure $P(r)$ associated with each value of water saturation.

### 2.3. Coupled elastoplastic model

For an elastic isotropic formation with a Biot coefficient of 1, stress equilibrium around a wellbore in a one-dimensional cylindrical system can be expressed as
\[
\frac{\partial \sigma'_t}{\partial r} + \frac{\sigma'_t - \sigma''_t}{r} = \frac{\partial P}{\partial r}
\]  \hspace{1cm} (16)

Also, the stresses within the plastic zone must fulfill the M-C failure criterion
\[
\sigma''_t = 2C_o \tan \beta + \sigma'_t \tan^2 \beta
\]  \hspace{1cm} (17)

Because capillary strength resulting from fluid menisci mainly prevents particle separation rather than rotation, it acts more as a part of the cohesive M-C strength component. Thus, the cohesive shear strength \(C_o\) can be expressed as
\[
C_o(S_w) = C_{o,\text{init}} - C_{o,\text{ch}} - \sigma_{\text{UCS}}(S_w) \tan \varphi
\]  \hspace{1cm} (18)

where \(C_{o,\text{init}}\) is the initial cohesive shear strength (before water breakthrough), and \(C_{o,\text{ch}}\) and \(\sigma_{\text{UCS}}\) are the reduced strengths that arise because of chemical reactions and capillarity.

The somewhat lengthy stress solutions can be found in a previous publication [19].

2.4. Nonlinearity modulus model

Besides effects on rock strength, water saturation changes can also lead to significant changes in the elastic properties. Young's modulus generally decreases with increase in water saturation [22-24], mimicking strength behavior in water-oil fluid environments, whereas Poisson's ratio may monotonously increase with saturation [12, 24] or remain constant [9], depending on rock type, mineralogy, and heterogeneity. In this research Poisson's ratio is assumed to remain constant.

Various nonlinear approaches have been developed to address stress-related nonlinear behavior in rock properties. They can be generalized into two categories: one is the nonlinearity due to shear damage caused by elevated shear stress [25, 26]
\[
E = A_E \left( 1 - \frac{R_f (1 - \sin \varphi) (\sigma'_t - \sigma'_s)}{2C_o \cos \varphi + 2\sigma'_s \sin \varphi} \right)^2
\]  \hspace{1cm} (19)

where \(R_f\) is a parameter accounting for the effect of residual strength after the stress reaches a peak value, and \(A_E\) is the Young's modulus at initial stress state. The other category is the nonlinearity due to the confining stress compaction effect [27]:
\[
E_i = E_a \left( 1 + m_E \sigma''_s \right)
\]  \hspace{1cm} (20)

where \(E_a\) is the rock Young's modulus at atmospheric pressure, and \(m_E\) and \(n_E\) are constants determined from curve fitting. This equation was shown to be reasonable through a series of experiments [28]; therefore it is used in this study to calculate \(A_E\) in Eq. (19).

3. CALCULATIONS AND DISCUSSIONS

Besides the assumptions made with respect to the microscopic capillarity model [16], it is necessary to clearly restate some other model simplifications and assumptions before any further discussion:

- Rock particles are represented by spheres;
- No residual water or oil saturation is considered in the capillarity model;
- Biphasic fluid flow is steady, and capillary pressure around the wellbore is related only to water saturation;
- Stresses are calculated around an axisymmetric vertical wellbore in a one-dimensional cylindrical system in an isotropic formation with Biot coefficient of 1;
- Rock behaves elastoplastically and strength in the plastic zone is a constant for stress calculations; and,
- Rock stress-strain curves fit a hyperbolic relation in the nonlinearity modulus model.

Whereas these may be viewed as limitations to the models' applicability, the authors believe that because the models capture the essential physics, adjustments and calibrations can be incorporated to allow useful applications in real situations.

3.1. Strength weakening

With particle radius of 0.1 mm and surface tension of 0.036 N/m, the maximum capillary strength can be as high as 20 kPa (Fig. 3), whereas all capillary variables become zero around a saturation value of 0.34. The magnitude of the capillary strength is closely related to particle radius, surface tension, liquid-solid contact angle, size difference and the distance between particles, and the irregularity of particle surfaces [18]. However, there is a small section of the relationship at a water saturation approaching zero where a short rapid increase of strength is predicted because some volume of water is needed to build a stable liquid bridge between particles. This has been confirmed by an experiment in which a stable arch is found to develop with a small increase in water saturation in
a biphasic fluid environment, whereas such an arch cannot be stable in a single-phase condition [6].

3.2. Fluid pressure fluctuations
For pressure and stress calculations, the water saturation in the microscopic model developed above should be calibrated to experimentally determined values. The saturation discrepancy between the model and reality results mainly from two sources that the microscopic model cannot address: one is connate water saturation \( (S_{wc}) \) and immobile oil saturation \( (S_{oi}) \); the other is the wettability effect for irregular particle surfaces. Assuming water saturation remains as a constant \( (S_{w0}) \) until water breakthrough occurs, the calibration can be carried out as

\[
S'_{w} = S_{wc} + S_{w0} \left( 1 - S_{wc} - S_{w0} \right) / S_{w0}
\]

where \( S_{w0} \) is the saturation at which capillary pressure becomes zero.

Using the relative permeability data in Table 1, calculations of pressure redistributions with water saturation at different distance \( (r = 0.2 \text{ m}, 1.0 \text{ m}, 2.55 \text{ m}) \) from the wellbore are plotted in Fig. 4. Interestingly, pore pressure first decreases with saturation until some critical saturation is reached \( (S_{w} = 0.45) \), and the decrease in magnitude can be as high as several MPa; then, it increases continuously to a value (when \( S_{w} = 0.734 \)) even higher than initially (when \( S_{w} = 0.32 \)). Furthermore, the pressure decrease becomes more significant closer to the wellbore, as illustrated by solid line in Fig. 4 (i.e. \( r = 0.2 \text{ m} \)). Physically, because water is a less viscous and more mobile fluid than oil, less energy is needed to drive it into the wellbore; consequently, the increase of water relative permeability raises the pore pressure whereas increase in oil relative permeability lowers it. The synthesis of both effects indicates that pore pressure in a water-dominant fluid system is relatively higher than in an oil-dominant fluid system.

The precision of pressure solutions from the new approach based on grain-scale physics is in the order of \( 10^{-15} \) comparing to the conventional method \( (P = P_{w}*S_{w} + P_{o}*S_{o}) \), precise enough to be applied in pressure and stress analyses.

3.3. Stresses redistributions with saturation
The parameters used in the stress model are listed in Table 2. Corresponding to the changes of rock strength and pore pressure, the effective normal stresses shift significantly away from the wellbore, along with an increase of the shear stress (Fig. 5). This leads to more rock that is more likely to experience shear failure in higher water saturation areas. Then, stresses move back towards the...
opening because of the increased pressure at later stages of water invasion. The stress increase reaches several MPa in magnitude.

3.4. Propagation of the plastic zone
Both strength reduction and increased stress have significant impacts on rock stability and behavior after water breakthrough. Stability influence is interpreted in terms of a critical radius ($R_c$) in Fig. 6 that defines the boundary between elastic and plastic zones around a wellbore. The solid lines describe the propagation of $R_c$ with saturation for sands with different initial cohesive shear strengths ($C_{o,\text{init}}$). Dimensionless critical radius is defined as the ratio of $R_c$ to wellbore radius $R_1$. Clearly, saturation has a large impact on the plastic yield zone: $R_c$ increases rapidly with the increase of saturation.

Even though it is hard to rigorously quantify the effects of chemical reactions, one qualitative way is to consider a reduced initial cohesive shear strength, as shown in Fig. 6. When the rock initial strength is decreased from 0.5 to 0.4 MPa because of the chemical reactions discussed above, the plastic radius increases around 2.5 times at the same saturation (e.g. $S_w = 0.45$). This indicates that strength-weakening chemical reactions can greatly affect the amount of failed rock around a wellbore.

Compared to the dashed lines that treat rock strength as a constant, i.e. no capillary strength appears and rock stability changes only result from pore pressure variations, the capillary effect is far less significant than the effect of relative permeabilities unless the initial rock strength is relatively low (e.g. $C_{o,\text{init}} = 0.4$ MPa). Considering that the magnitude of capillary strength (on the order of kPa) is much lower than rock strength (on the order of MPa), this defines when capillary strength plays an important role in stabilizing sand: after most of its initial strength has been destroyed due to high shear stress or lost due to chemical reactions. Only at this stage can the effect of capillarity become a significant factor.

Furthermore, after shear failure, the only possible cohesive bond existing among sand particles is capillary tensile strength. Therefore capillarity plays a dominant role in the final phase of erosion and transport of failed sands into the wellbore. Since capillary strength is shown to depend only on water saturation, the sanding rate for each value of saturation becomes constant until either the cohesive strength or the destabilizing forces, e.g. fluid seepage force and loading force resulting from shear stresses, are changed, which leads to so-called episodic sand production after an oil well starts to produce water [5].

![Fig. 5. Variations of effective stresses with saturation](image1)

![Fig. 6. Effect of capillary strength on plastic yield front](image2)

**Table 2. Parameters used in the geomechanical (stress) model**

<table>
<thead>
<tr>
<th>Rock Mechanical Properties</th>
<th>Reservoir Flow Properties</th>
<th>Geometry Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ (Pa)</td>
<td>$n$</td>
<td>$R_2$ (m)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>$\sigma_h$ (Pa)</td>
<td>$R_1$ (m)</td>
</tr>
<tr>
<td>$C_o$ (Pa)</td>
<td>$\phi$</td>
<td>$h$ (m)</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>$k_i$ ($m^2$)</td>
<td></td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>$P_2$ (Pa)</td>
<td></td>
</tr>
<tr>
<td>$0.3$</td>
<td>$\mu$ (Pa/s)</td>
<td></td>
</tr>
<tr>
<td>$3 \times 10^9$</td>
<td>$10 \times 10^6$</td>
<td>$1.157 \times 10^{-3}$</td>
</tr>
<tr>
<td>$0.45$</td>
<td>$0.3 \times 10^{-12}$</td>
<td></td>
</tr>
<tr>
<td>$28 \times 10^6$</td>
<td>$0.5 \times 10^6$</td>
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<tr>
<td>$30^\circ$</td>
<td>$10 \times 10^6$</td>
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<tr>
<td>$0.3$</td>
<td>$0.01$</td>
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<tr>
<td>$3 \times 10^9$</td>
<td>$10 \times 10^6$</td>
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<td>$0.45$</td>
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<td>$30^\circ$</td>
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<td>$30^\circ$</td>
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<td>$0.5$</td>
<td>$0.01$</td>
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</tbody>
</table>
3.5. Rock softening

To investigate rock nonlinearities in stressed biphasic environments, both stress variables ($\sigma'_o$ and $\sigma'_r$) from the stress model and water-related rock strength ($C_o$) from the strength model serve as inputs to Eqs. (19) and (20).

Corresponding to shear stress redistribution that significantly changes with water breakthrough (Fig. 7), the Young’s modulus decreases from 2.3 GPa to 1.3 GPa, a loss of about 45%, before it regains part of stiffness because of pressure recovery and therefore effective stress release. Furthermore, the magnitude of modulus loss and stress increase with water saturation depends on location in the rock and its initial strength: the farther the rock is located from the well ($r/R_1 = 7.5, 15$) and the stronger it is ($0.4, 0.45, 0.5$ MPa), the less the modulus loss and the stress increase. This confirms experimental observations that weaker rock is more sensitive to changes in moisture content [29].

4. CONCLUSIONS: WHY SAND FAILS AFTER WATER BREAKTHROUGH

Analytical research and model development has generated a new rock strength model to capture the effects of capillarity and chemical reactions, a new fluid pressure model based on micromechanics, and a coupled analytical elastoplastic model for stress estimations around an oil well. These efforts have helped to identify and study sand instability mechanisms so that the question of why sand fails after water intrusion can be more logically approached.

In summary, with an increase in water saturation, sands tend to become weaker (strength reduction) and softer (stiffness reduction), while loading stresses (effective stress and shear stresses) are elevated and the maximum shear stress moves outward into the reservoir, affecting a larger volume of rock. Consequently this rock is more likely to experience shear failure that destroys or damages the cohesive or interlocked fabric. Furthermore, weakened sands are more easily detached from the rock matrix because of the decrease of the tensile capillary strength with an increase in water saturation. Since the capillary strength only depends on water saturation if the rock and fluid properties are fixed, the sanding rate for each saturation value should be constant until the destabilizing forces are changed, which leads to the observed episodic sand production that develops after water breakthrough.

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NOMENCLATURE AND UNITS

- $C_o$: cohesive strength of the rock, Pa
- $C_o_{\text{init}}$: initial cohesive strength, Pa
- $C_{o,ch}$: cohesive strength affected by chemical reactions, Pa
- $E$: Young’s modulus, Pa
- $f_w, f_o$: water and oil production fraction, dimensionless
- $F_c$: capillary bond force, N
- $k$: absolute permeability of reservoir, m$^2$
- $k_{rw}, k_{ro}$: water and oil relative permeabilities, m$^2$
- $m_{E}, n_E$: coefficients for modulus nonlinearity, dimensionless
- $\Delta P$: capillary pressure, Pa
- $P_w, P_o$: water and oil pressures, Pa
- $P_2$: far-field reservoir pressure, Pa
- $Q$: fluid production, m$^3$/s
- $Q_w, Q_o$: water and oil production rates, m$^3$/s
- $r$: radius of liquid bridge between particles, m
- $R$: radius of the particles, m
- $S_w$: water saturation, dimensionless
- $S_{wC}$: connate water saturation, dimensionless
- $S_{oi}$: immobile oil saturation, dimensionless
- $S_{w0}$: the saturation at which capillary pressure becomes zero, dimensionless

Fig. 7. Young’s modulus vs. water saturation
\( \alpha \) volume angle of wetting fluid, radian
\( \beta \) failure angle \( (= \pi/4 + \varphi/2) \), radian
\( \gamma \) surface tension between two fluids, N/m
\( \varphi \) internal friction angle defined in Mohr-Coulomb, degree
\( \phi \) rock porosity, dimensionless
\( \eta \) the coefficient defined to balance porosity difference between the model and reality, dimensionless
\( \sigma_{\tau}, \sigma'_{\tau} \) effective radial and tangential stresses, Pa
\( \sigma_{T,C} \) capillary tensile strength of the rock, Pa
\( \sigma_{\text{UCS}} \) Uniaxial Compressive Strength, Pa
\( \mu_w, \mu_o \) oil and water viscosity, Pas
\( \nu \) Poisson’s ratio, dimensionless

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